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EDITORIAL

Focus on stochastic flows and climate statistics

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Abstract

The atmosphere and ocean are examples of dynamical systems that evolve in accordance with the laws of physics. Therefore, climate science is a branch of physics that is just as valid and important as the more traditional branches, which include particle physics, condensed-matter physics, and statistical mechanics. This 'focus on' collection of *New Journal of Physics* brings together original research articles from leading groups that advance our understanding of the physics of climate. Areas of climate science that can particularly benefit from input by physicists are emphasised. The collection brings together articles on stochastic models, turbulence, quasi-linear approximations, climate statistics, statistical mechanics of atmospheres and oceans, jet formation, and reduced-form climate models. The hope is that the issue will encourage more physicists to think about the climate problem.

1. Introduction

The atmosphere and ocean are examples of dynamical systems that evolve in accordance with the laws of physics [1]. The relevant branches of physics include classical fluid dynamics, thermodynamics, and statistical mechanics. Motions are energised on length scales ranging from the planetary scale of thousands of kilometres to the Kolmogorov scale of a few millimetres or less. This is a vast range that encompasses many orders of mangitude [2]. The dimensionality of the dynamics of the atmosphere and ocean, as measured by the number of independent degrees of freedom, is therefore very high. The atmosphere and ocean are forced on the planetary scale by incoming solar radiation and its uneven distribution with latitude. Energy cascades downscale in three-dimensional turbulent motions and is ultimately dissipated at the Kolmogorov scale by the effects of molecular viscosity. In addition to the atmosphere and ocean, the global climate system is also comprised of the hydrosphere, cryosphere, lithosphere, and biosphere together with their complex interactions and feedbacks.

Given the very high dimensionality of the climate system, any attempt to understand, model, and predict it will inevitably involve some degree of approximation and simplification. This statement applies to conceptual, toy models with only a few degrees of freedom, but it also applies to the comprehensive general circulation models of the atmosphere and ocean, which solve either finite-difference or truncated spectral representations of the governing partial differential equations. As a consequence, only the gravest modes of variability, representing the largest length scales and longest time scales, are explicitly resolved. The remaining degrees of freedom are either neglected or approximately represented using closure schemes known as parameterisations. The idea that the climate system can be separated into slow and fast components, with the fast components being represented as stochastic noise, dates back to the 1970s [3].

One manner in which stochastic approaches can enter into climate science is by 'stochasticising' the parameterisation schemes, which have conventionally been deterministic. This approach has been gaining popularity recently, as discussed in several review articles [4–7]. The question of whether stochastic closure schemes are better than deterministic closure schemes has been listed as an outstanding challenge in the area of mathematics applied to the climate system [8]. One benefit that may be conferred by stochastic

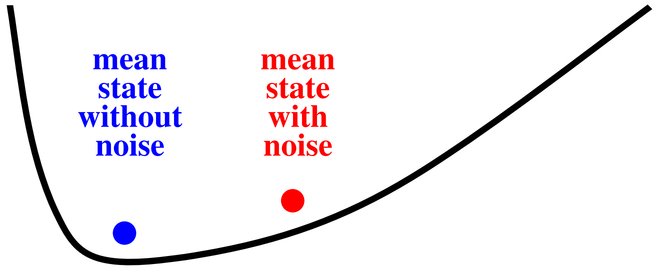

Figure 1. Schematic representation of the phenomenon of noise-induced drift. The state of the system is represented by the horizontal position of a ball moving in an asymmetric potential well. Without external noise, the ball equilibrates at the minimum of the potential well. In contrast, when evolving in the presence of external noise, the average position of the ball is on the shallow side of the minimum. Therefore, the mean state of the system is modified by the noise.

parameterisations is improvements to the mean state of the climate via the phenomenon of noise-induced drift, as shown schematically in figure 1. Indeed, such an effect has now been demonstrated in climate simulations using comprehensive coupled atmosphere–ocean general circulation models [9]. Another impact of stochastic parameterisations is the possibility of noise-induced regime transitions, which have now been observed in laboratory experiments on rotating fluids [10, 11] and have led to a new interpretation of polar vortex splits [12].

This 'focus on' collection of *New Journal of Physics* brings together original research articles from leading groups that advance our understanding of the physics of climate. Areas of climate science that can particularly benefit from input by physicists are emphasised. The collection brings together articles on stochastic models, turbulence, quasi-linear approximations, climate statistics, statistical mechanics of atmospheres and oceans, jet formation, and reduced-form climate models. We hope that the issue will encourage more physicists to think about the climate problem.

2. Contents of this special collection

In broad terms, three of the articles in this special collection focus on stochastic models. The first is a study by Jeffress and Haine [13] on estimating sea-surface temperature transport fields from stochastically forced fluctuations. Previous time-lag correlation methods for quantifying the transport of sea-surface temperature have neglected diffusion and relaxation to atmospheric temperatures. The new study quantifies the transport more completely by estimating a response function using a fluctuation–dissipation approach. This method accounts for all of the physical mechanisms involved in the transport, by including diffusion and relaxation in addition to advection. Using 100 years of data from a stochastically forced prototypical model, it is shown that the method estimates the transport response function to within an error of 10%. The method has the potential to provide independent estimates of sea-surface temperature (and also salinity) transports, for comparison with previous studies.

The second study to focus on stochastic models is by Moon and Wettlaufer [14] and discusses the interpretation of Stratonovich calculus. The fact that there are two different meaningful interpretations of stochastic calculus is a constant source of confusion to climate scientists who are encountering stochastic approaches for the first time. Loosely speaking, if a system is being forced by continuously fluctuating noise with finite memory, then the Stratonovich interpretation is appropriate. On the other hand, if a system is being forced by random discrete pulses, then the Itô interpretation is appropriate. The Stratonovich integral obeys the usual chain rule, whereas the Itô integral does not. The Stratonovich interpretation is more common in physics and engineering, whereas the Itô interpretation is more common in mathematics. If noise is additive then both interpretations lead to the same Fokker–Planck equation, but if it is multiplicative then the interpretations can give different results. By considering the finite decay of the noise correlations, the new study in this special collection suggests a generalisation of an integral Taylor expansion criterion for the validity of the Stratonovich approach.

The third study to touch on stochastic models is by Restrepo *et al* [15] and considers methodologies for defining a trend from a given time series. This is a topic of obvious importance in climate change. The extraction of a trend is complicated by the fact that the observed global temperature record contains variability on a wide spectrum of time scales, in addition to the long-term anthropogenic trend that is of interest. The new study proposes a new method for defining a trend, or tendency, for time series with inherent multi-scale features. The method involves first using an intrinsic time-scale decomposition to strip out the random noise (or high-frequency variability) from a time series and produce a set of candidate tendencies. The method then applies a particular criterion to each of the candidates, in order to single out the best one. The method is tested by applying it to ocean temperatures as well as atmospheric temperatures in Arizona and Moscow.

Numerical simulations of geophysical fluids suffer from the 'curse of dimensionality' [16, 17]: only a small fraction of the active degrees of freedom can be captured by a computer. Weidauer and Schumacher [18] search for, and find, reduced descriptions of shallow moist convection such as occurs in the cloud-topped atmospheric boundary layer. They employ the method of 'proper orthogonal decomposition' to identify the important modes, and find it possible to reduce by more than two orders of magnitude the number of degrees of freedom.

Edward Lorenz observed, nearly 50 years ago: 'More than any other theoretical procedure, numerical integration is also subject to the criticism that it yields little insight into the problem. The computed numbers are not only processed like data but they look like data, and a study of them may be no more enlightening than a study of real meteorological observations. An alternative procedure which does not suffer this disadvantage consists of deriving a new system of equations whose unknowns are the statistics themselves' [19]. Eschewing the traditional route of accumulating statistics from the time series from numerical simulation, three of the articles in the collection focus on directly accessing the statistics of idealised climate models. The first study is by Parker and Krommes [20] and examines in detail the process by which jets, or zonal flows, develop out of homogeneous turbulence in the context of an idealised stochastically driven barotropic model. The work builds upon the Stochastic Structural Stability Theory (S3T) of Farrell and Ioannou [21], and the method of cumulant expansion [22], both of which are closures enabled by a quasi-linear approximation [23] in which interactions that are purely between eddies are dropped. Parker and Krommes study how jets emerge from turbulence, spontaneous breaking translational symmetry in latitude.

Deterministic models are also amenable to direct statistical simulation. Chaalal *et al* [24] apply low-order cumulant expansions to two models: a barotropic model of wave breaking, and a model of the dry convective boundary layer. They find that a closure at second order in the cumulants suffices to capture some important features of the two models. In particular it can capture the deepening of the boundary layer but not the turbulent transport of kinetic energy. For evolution of the barotropic wave they find that the second-order closure works well when the waves are weak. However, unsurprisingly, a cumulant expansion of the second order fails to capture the flow evolution when strongly nonlinear eddy–eddy interactions are important. Improved approximations that go beyond the quasi-linear approximation appear necessary, and Chaalal *et al* demonstrate that one such improvement works.

Expansions in cumulants are limited to low-order statistics of the flows. Laurie and Bouchet [25] investigate instead high-order statistics. They examine barotropic flows that make rare transitions between two equilibria, applying instanton methods (as they are known to physicists) or equivalently large-deviation theory (the term preferred by mathematicians). Some real geophysical flows exhibit such transitions—the splitting of the polar vortex in the northern stratosphere [12] is one prominent example—and it may be possible to apply the methods developed by Laurie and Bouchet to more realistic models of atmospheric and oceanic fluid dynamics to capture rare events that are difficult to simulate directly by computer.

Many flows in nature do not display the scale invariance of fully developed hydrodynamic turbulence, because they are in a regime of either spatio–temporal chaos or weak turbulence. An example is Rayleigh–Bénard convection with Prandtl number near one. This system exhibits bi-stability between ideal straight convection rolls and weak turbulence in the form of spiral defect chaos. Schütz and Bodenschatz [26] investigate Rayleigh–Bénard convection in this volume through the lens of mass transport, by identifying and partially explaining three scaling regimes for the dispersion of particles in numerical simulations. Rather surprisingly, diffusive-like spreading of particles is found at long time-scales, even in the absence of turbulence. The spreading rate is found to depend on the degree of spatio–temporal chaos. Larger correlation lengths require longer observations and larger system sizes, meaning that the computational expense rapidly becomes prohibitive, and leaving several open questions for future work.

3. Outlook

The papers that are contained in this special issue are just a small sample of the work that is currently being undertaken internationally on the topic of stochastic flows and climate statistics. Mindful of the pressing societal importance of gaining a deep understanding the climate system, our hope in editing this special collection is to inspire future research directions and to spur on future studies. The work presented here raises more questions than it answers. To take one example: the 'curse of dimensionality' reappears with a vengeance in statistical formulations of geophysical fluid dynamics. Will it be possible to find efficient reduced descriptions for numerical simulations, as Weidauer and Schumacher [18] do?

A recent review of numerical weather and climate prediction [27] observes that 'the uncertainties inherent to physical parameterisations, either from incomplete process understanding or the dilemma of representing the impact of unresolved processes on the resolved scales, may require a fundamentally different approach. Elements of parameterisations or entire schemes are likely to require components that appear statistical to the large scales because they are not fully determined by the resolved scales. Examples are stochastic sampling of parameter probability distribution functions, stochastically driven sub-cell models, or super-parameterisations through embedding entire convection-resolving simulations at sub-grid scale. How radical this approach needs to be is currently not clear'. The present collection may represent one step in the direction of answering this important question.

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